VII. *Elements of Captain* HALL'S *Comet. By* J. BRINKLEY, *D.D. F.R.S. and M.R.I.A. and Andrew's Professor of Astronomy in the University of Dublin. In a Letter addressed to* W. H. WOLLASTON, *M.D. V.P.R.S.*

Read, January 10, 1822.

Observatory, Trinity College, Dublin,
October 15, 1821.

MY DEAR SIR,

I SEND you the elements of the comet observed at Valparaiso, the observations of which you were so kind as to send to me.

We are indebted to the science of Captain HALL, for adding this comet to our catalogue.

The observations appear to have been as exact as could have been made even in an established observatory, on a comet only visible so near the horizon, and so far from the meridian, and of which the light was probably faint both from its actual distance from us, and its apparent proximity to the sun.

The comet on the 8th of April was distant nearly 1,41 from the earth, the sun's distance from the earth being unity, and on the 3d of May, when last seen, about 2,64.

This comet is interesting to astronomers on account of its small perihelion distance. In the Catalogue of M. DELAMBRE, out of 116 comets, the orbits of which have been computed, there are only three that pass nearer the sun.

It is interesting also, from the probability that it is the same comet that was observed in 1593, which agrees with this in

its small perihelion distance and great inclination. The inclination was computed by LACAILLE to be 88° nearly. The inclination of this is 106° 44′. The perihelion distance of that = ,089, of this ,093.

This comet would probably have been a very remarkable one by its appearance, had the earth been in a favourable situation.

It may, at first, cause some surprize, that it escaped the watchful observers of Europe when on the north side of the ecliptic, in February and March last, before it passed its perihelion; but on examination it will be found, that in March it was only a few degrees from the sun, and must have been rendered invisible by the superior lustre thereof. It passed from the north to the south side of the plane of the earth's orbit, March 21st, $18^h\ 30^m$, a few hours after it had passed its perihelion; and its elongation from the sun then was only about 4°.

The observations having been made so long after its passage through perihelion, the smallness of its perihelion distance, and some other circumstances, have involved the computation in difficulties not often met with. This induces me to request you will lay before the Royal Society, the method by which I proceeded.

The unusual circumstances relative to this comet, will also appear by referring to a passage in the Preface of M. BURCKHARDT'S new table of the Parabolic Movements, &c. in the Conn. des Temps, 1818. It relates to the interval of his table from 1000 to 10,000 days, of which he says, " comme il est probable que cette partie ne servira *jamais* dans la pratique, &c." Now, in the observations after the 18th of April,

the anomalies of this comet exceeded that of 1000 days in the tabular comet.

Besides the elements (B) deduced by the method of which the explanation is given, I had previously deduced another set (A) by a process less certain in its operations. The latter set (A) however, may be thought not much inferior to the other set (B).

I remain, my dear Sir,

your's most truly,

Dr. Wollaston, &c. &c. &c. J. BRINKLEY.

Elements (B.)

Perihelion distance ,092800

Time of passage through perihelion

Mean Time, Greenwich, March 21st. - - - h. m. s. 13 15 47

		°	′	″
Inclination - -	s.	73	15	48
Node (ascending)	1	18	24	41
Perihelion - -	7	29	6	47

Motion retrograde.

Elements (A.)

Perihelion distance ,0894

Passage through perihelion March 21, - - - h. m. s. 7 3 26

		°	′	″
Inclination - -	s.	74	32	41
Node - - -	1	19	38	17
Perihelion - -	8	0	35	8

Motion retrograde.

	Observation.	Observation.	Observation.	Error of Elements (B.)		Error of Elements (A.)	
1821.	Mean Time Valparaiso.	Longitude.	Latitude S.	Long.	Lat.	Long.	Lat.
	h. m. s.	° ′ ″	° ′ ″	′ ″	° ″	′ ″	′ ″
April 8	7 10 58	33 27 44,3	21 48 8,0	0 0	0 0	+0 20	0 0
11	6 54 45	36 47 3,4	22 10 15,2	+1 28	+0 29	+0 53	+0 2
12	7 2 30	37 49 5	22 15 26	+1 11	+0 45	+0 26	+0 13
14	6 54 0	39 44 25,9	22 24 26,2	+0 56	+0 7	+0 1	—0 29
17	7 0 29	42 21 57	22 33 9	+0 56	—0 30	0 0	—1 7
18	6 36 52	43 9 14	22 34 7	+1 36	+0 17	+0 45	—0 18
19	6 34 54	43 57 55	22 34 55	+0 1	+0 56	—0 46	+0 24
20	6 28 19	44 42 53	22 36 50	+0 25	+0 9	—0 14	—0 21
21	6 30 10	45 26 57,0	22 37 58,8	+0 36	—0 8	+0 4	—0 34
24	6 49 30	47 31 42,7	22 41 31,3	+1 11	—2 15	+1 6	—2 32
29	6 48 36	50 41 8,7	22 39 37,2	—2 4	—0 38	—1 16	—0 41
May 1	6 31 40	51 44 55,1	22 37 45,6	+2 4	+0 51	+3 13	+0 49
3	6 29 37	52 52 55,1	22 37 42,7	—0 50	+0 15	+0 45	+0 22

Computation of the Elements.

The method used for the first approximation was that of M. LAPLACE.* The observations chosen were those of April 8, 11, and 14. It considerably diminishes the length of the calculation to use only three observations, where it can be done. In the present instance it is not likely, had a greater number of observations been used, that the superior accuracy derived would have compensated for the additional trouble. The result from the three observations, is as exact as would be necessary on common occasions. Here, on account of the small perihelion distance and small motion of the comet in heliocentric longitude between the first and last observations, the method of M. LAPLACE,† "Determination exacte des elemens de l'orbite, &c. &c." is inapplicable; the errors of the motion in heliocentric longitude being greater than the motion itself.

The extension however of that method, as indicated by M. DELAMBRE,‡ might succeed in this instance, but the formidable calculations necessary are sufficient to deter one from the attempt.

The method which I gave in Vol. XIII. p. 189, of the Transactions of the Royal Irish Academy, also, in this case, requires to be extended. It will be necessary to introduce the squares of the variation of the perihelion distance, and of the correction of the time from perihelion.

The manner in which my method is extended to this case will be easily understood from the following detail; and also the change I have made in deriving on common

* Mec. Celes. Tom. i. p. 221, &c. † p. 225. ‡ Astron. Tom. 3. p. 384.

occasions the final equations, by using a formula for the computation of V, different from that given by M. LAPLACE.

Previously to the computation, I re-calculated the observations of April 8, 11, 14, 21, and May 3, using the places of the stars as given by M. PIAZZI. This perhaps was unnecessary.

The results were

	Æ	Declination S.
	h. m. s.	° ′ ″
April 8	2 34 15,0	7 51 52
11	2 46 28,0	7 12 4
14	2 57 14,2	6 33 51
21	3 17 46,1	5 13 35
May 3	3 44 19,7	3 25 55

By M. LAPLACE's method, " Determination approchée, &c." the observations of April 8, 11, and 14, give perihelion dist. $(p) = ,0865$, and time of perihelion, March $19^d\ 14^h\ 4^m$.

Let the true perihelion dist. $= p + dp$, and time of perihelion $=$ March $19^d\ 14^h\ 4^m - dt$, so that t being the interval between March $19^d\ 14^h\ 4^m$, and the observation of April 8, the true value of $t = t + dt$.

Let also T $=$ the time in the table of the comet of 109 days, when the anomaly $= v$.

$\Delta =$ variation of anomaly in that table in one day at time T, and $r =$ comet's distance from the sun

$$\text{Then } dv = \left(\frac{dt}{p^{\frac{3}{2}}} - \frac{3tdp}{p^{\frac{5}{2}}}\right)\Delta \quad - \quad - \qquad (1)$$

$$\text{and } (x)\ \frac{d\log r}{\sin 1''} = dv \tan \tfrac{1}{2} v + \frac{dp}{p \sin 1''} \qquad (2)$$

Conceive the triangle in which S, T, C, represent the sun, earth, and comet respectively, and let P represent the projection of the comet on the plane of the earth's orbit.

In this case, comet's hel. long.$=$earth's long.$-$TSP. Also

d hel. lat. $(\pi) = x \tan SCT \cot CST \tan \pi$. - - (3)

d hel. long. $(\beta) = -x \cot TSP \tan SCT (\tan CST - \cot CST \tan^2 \pi)$ — (4)

These angles SCT, &c. are found in the course of the calculation.

If β, β' β'' represent three hel. longitudes, and π, π' π'' represent three hel. latitudes, and V, V′ represent the bases of two spherical triangles of which the sides are $90^\circ - \pi$, $90^\circ - \pi'$, and $90^\circ - \pi$, $90 - \pi''$, and the vertical angles $\beta - \beta'$ and $\beta - \beta''$ respectively,

$$\cos V = \cos (\pi - \pi') - 2 \sin^2 \tfrac{1}{2} (\beta - \beta') \cos \pi' \cos \pi \quad (5)$$

$$\cos V = \cos (\pi - \pi'') - 2 \sin^2 \tfrac{1}{2} (\beta - \beta'') \cos \pi'' \cos \pi \quad (6)$$

Taking the observations of April 8, 21, and May 3, and the above values of p and t,

	Anomalies.	Hel. Long.	Hel. Lat. S.
	° ′ ″	S. ° ′ ″	° ′ ″
April 8	139 46 38*$=\nu$	2 5 17 1$=\beta$	48 55 19$=\pi$
21	146 38 6 $=\nu'$	2 3 47 40$=\beta'$	42 43 42$=\pi'$
May 3	150 12 10 $=\nu''$	2 2 35 30$=\beta'$	39 22 50$=\pi''$

If $U = \nu' - \nu$ and $U' = \nu'' - \nu$

The observations of April 8 and April 21, give

$$U = 6^\circ\ 51'\ 28'' \quad V = 6^\circ\ 16'\ 44''$$

Those of April 8, and May 3, give

$$U' = 10^\circ\ 25'\ 32'' \quad V' = 9^\circ\ 44'\ 12''$$

Therefore because $U + dU = V + dV$

and $U' + dU' = V' + dV'$

* The results are put down to seconds; but as only five places in the logarithms were used in this part of the process, they may sometimes err by several seconds.

$$dV - dU = 2084'' \quad - \quad - \quad (8)$$
$$dV' - dU = 2480'' \quad - \quad - \quad (9)$$

By the formulæ above given

$$dv = 2751\, dt - 950060\, dp$$
$$dv' = 1336\, dt - 762080\, dp$$
$$dv'' = 861\, dt - 669980\, dp$$
$$x = 7512\, dt - 209300\, dp$$
$$x' = 4459\, dt - 158300\, dp$$
$$x'' = 3235\, dt - 133450\, dp$$
$$d\pi = -\,,4390\, x \quad .. \quad d\beta = -\,,9345\, x$$
$$d\pi' = -\,,3446\, x' \quad .. \quad d\beta' = -\,,4367 x'$$
$$d\pi'' = -\,,2954\, x'' \quad .. \quad d\beta'' = -\,,2130\, x''$$

From these values it is necessary to compute dV, dV'.

Let $H = 2 \sin^2 \frac{1}{2} (\beta - \beta') \cos \pi' \cos \pi$

$H = 2 \sin^2 \frac{1}{2} (\beta - \beta'') \cos \pi'' \cos \pi$

then equations (5) and (6) become

$$\cos V = \cos (\pi - \pi') - H \quad - \quad (10)$$
$$\cos V' = \cos (\pi - \pi'') - H' \quad - \quad (11)$$

It is obvious from the smallness of $\beta - \beta'$ and $\beta - \beta''$, and the magnitude of the errors of $U - V$ and $U' - V''$ that the errors of $\beta - \beta'$ and $\beta - \beta''$ may bear a considerable proportion to the quantities themselves, or be even greater. Therefore the coefficients of the differentials dp and dt in dH will be quite incorrect, if the differential of H be computed in the common way.

It is the smallness of $\beta - \beta'$ and $\beta' - \beta''$ in this case that renders M. LAPLACE's method of computing V, V′ inconvenient, in consequence of its being necessary to use the tangent of an angle nearly $= 90^\circ$.

But the variations of V and V′ may be obtained with sufficient accuracy in the following manner.

By equation (10) we easily deduce an account of the smallness of H

$$V = \pi - \pi' + \frac{H}{\sin 1'' \sin (\pi - \pi')}$$

and with sufficient exactness

$$V + dV = \pi - \pi' + d\pi - d\pi' + \frac{2 \sin 1'' \cos \pi' \cos \pi \, (\beta + d\beta - \beta' - d\beta')^2}{\sin (\pi - \pi')}$$

therefore

$$dV = -(d\pi - d\pi') + \pi - \pi' - V + \frac{2 \sin 1'' \cos \pi' \cos \pi \, (\beta + d\beta - \beta' - d\beta)^2}{\sin (\pi - \pi')}$$

and the same expression serves for dV' changing π', β' and V into π'', β'' and V'.

Hence, substituting the values of $\beta, \beta', \beta'', \pi', \pi'', \pi''$ &c. &c. we obtain

$$dV = -344^* + ,3446x' - 4390x + N\,(2680 - ,4672x + ,2188x')^2$$
$$dV' = -703 + ,2954x'' - 4390x + N'(4845 - ,4672x + ,1065x'')^2$$

where log N $= 5.63801$ and log N' $= 5.47328$

and finally, equations (8) and (9) give

$$N\,(2680 - 2534dt + 63150dp)^2 - 364dt - 150649dp = 2428 \quad (12)$$
$$N'(4845 - 3166dt + 83572dp)^2 - 452dt - 227619dp = 3212 \quad (13)$$

From these equations the values of dp and dt may be derived. The indirect solution seems to be the shortest, as we know the value of dp within narrow limits.

The first error of p as deduced from the approximation of LAPLACE, rarely indeed will amount to $\pm$,01.

1. Let us suppose $dp = -$,005. Then equation (12) gives $dt = +4,294$ and $-1,194$. The positive value is too great to be admitted, taking $t = -1,194$, the result from equation (13) is $0 = -468$.

* This number $344'' = 6^\circ\ 16'\ 44'' - 6^\circ\ 11'$ should be strictly $367'' = 6^\circ\ 16'\ 44'' - 6^\circ\ 11'\ 37''$; but the accidental omission of $37''$ is not of the smallest consequence in the result.

2. $dp = ,000$

$dt = +4,901$ and $-1,547$, substituting in equation (13) $t = -1,547$

$o = -309$

3. $dp = +,005$

$dt = +5,442$ and $-1,838$

$o = -137$

the value of dp therefore exceeds $+,005$.

The last result shows, that with this value of dp, the errors of $V - U$ and $V' - U'$ are reduced to comparatively small quantities, and it may be useless in this first process to endeavour to reduce them still more.

Therefore we may now make $p = ,0865 + ,005 = ,0915$, and $t = 19,9118 - 1,838 = 18,0738$.

With these values we obtain, having corrected the sun's places and comet's observed places for aberration, and made the computations to seven places of figures,

	° ′ ″	s ° ′ ″	° ′ ″
April 8	136 53 51 $= \nu$	2 9 47 32 $= \beta$	50 51 16 $= \pi$
21	144 50 48 $= \nu'$	2 4 59 14 $= \beta'$	43 38 4 $= \pi'$
May 3	148 48 45 $= \nu''$	2 3 12 5 $= \beta''$	39 56 29 $= \pi''$

$V = 7° 55' 6''$, $V' = 11° 53' 33''$.

On account of the great changes that have taken place in some of these quantities, particularly in the longitudes; it will be better to re-compute $d\nu$, $d\pi$, $d\beta$, &c. &c.

This in common cases would be unnecessary.

The new values are

$$dv = 3299\,dt - 977400\,dp$$
$$dv' = 1510\,dt - 768635\,dp$$
$$dv'' = 947\,dt - 667980\,dp$$
$$x = 8353\,dt - 220500\,dp$$
$$x' = 4768\,dt - 172700\,dp$$
$$x'' = 3390\,dt - 138150\,dp$$
$$d\pi = -,4543x \;.\; d\beta = -1,1466\,x$$
$$d\pi' = -,3599x' \;.\; d\beta' = -,4756\,x'$$
$$d\pi'' = -,3053x'' \;.\; d\beta'' = -,2252\,x''$$

We may now use the equations (10) and (11) for finding dV and dV'

$dV \sin V = \sin(\pi - \pi')\,(d\pi - d\pi) + \frac{dH}{\sin 1''}$

$dH = H \sin 1''(\tan\frac{1}{2}(\beta - \beta')\,(d\beta - d\beta') - d\pi' \tan\pi' - d\pi \tan\pi)$

The expressions for dV' and dH' are had by changing H, β' and π' into H$'$, β'' and π''.

Then, by substituting the above values for $d\pi$, $d\pi''$, &c. &c.

$$dV = -3858\,dt + 80110\,dp$$
$$dV' = -4888\,dt + 112190\,dp$$

and the equations

$$U + dU = V + dV$$
$$U' + dU' = V' + dV$$

become

$$-2069\,dt - 128650\,dp = 111 \quad - \quad (14)$$
$$-2536\,dt - 197230\,dp = 80 \quad - \quad (15)$$

which give

$$dp = +,001425 \text{ and } dt = -,1423$$

Consequently, the new approximate values of p and t are

$$p = ,092925 \text{ and } t = 17,9315$$

Repeating the computation with these values we obtain

	° ′ ″	s ° ′ ″	° ′ ″
April 8	136 22 43 $=\nu$	2 10 16 43 $=\beta$	51 2 44 $=\pi$
21	144 28 59 $=\nu'$	2 5 6 34 $=\beta'$	43 43 35 $=\pi'$
May 3	148 30 40 $=\nu''$	2 3 3 35 $=\beta''$	39 59 56 $=\pi''$

$$V = 8^\circ\ 6'\ 29 \qquad V' = 12^\circ\ 8'\ 7''$$

and thence $U - V = -13''$ and $U' - V' = -11$
Substituting these values on the right hand side of equations (14) and (15), and solving the equations

$$dp = -,0001249 \text{ and } dt = ,014049$$

and the new values of p and t are $p = ,092800$, and $t = 17,9456$, which are sufficiently exact. From these new values of dp and dt we very easily get from the above values of $d\nu$ &c. &c.

	° ′ ″	s ° ′ ″	° ′ ″
April 8	136 25 33 $=\nu$	2 10 13 57 $=\beta$	51 1 39 $=\pi$
21	144 30 56 $=\nu'$	2 5 5 52 $=\beta'$	43 43 3 $=\pi'$
May 3	148 32 16 $=\nu''$	2 3 3 20 $=\beta''$	39 59 36 $=\pi''$

$$V = 8^\circ\ 5'\ 24'' \quad V' = 12^\circ\ 6'\ 45''$$

The differences between the first values of β, β' &c. &c. and these correct values, seem deserving of notice.

It remains to find the place of the node, inclination of the orbit, and the place of perihelion.

For this purpose it is very convenient and sufficiently exact* to compute, in the spherical triangle formed by the sides

* We are enabled to use this short process for finding the inclination, in consequence of being able to compute so readily the new values of dV, and dV as well as of $d\pi$, $d\beta$, &c. &c. by substituting in the values above given, the last values of dp, dt. This may be considered as another advantage of this method of correcting the approximate elements of a comet's orbit.

$90-\pi$, $90-\pi''$ and V′, the angle opposite to $90-\pi''$, and also in the spherical triangle formed by $90-\pi$, $90-\pi'$ and V, the angle opposite to $90-\pi'$.

These angles are

$152^\circ\ 38'\ 9''$, which gives the inclination $73^\circ\ 19'\ 50''$

$152\ \ 51\ \ 56$, which gives the inclination $73\ \ 11\ \ 47$

Now, if the orbit were an exact parabola, and the observations perfectly accurate, the above quantities ought, in consequence of the exactness that has been used in the computation, to agree to a second. Hence is seen the unavoidable errors arising from these two causes jointly. It is probable, that the deviation from a parabola has no sensible effect in the present case.

With the mean value $152^\circ\ 45'\ 3''$ we get the place of the node $48^\circ\ 24'\ 41''$, and the place of perihelion $=7^s\ 29^\circ\ 6'\ 47''$.

There evidently appears an irregularity in the three last right ascensions observed April 29, May 1 and 3. It is possible, that this happened in consequence of the difficulty of observing the comet from its faintness: if so, each of the observations may partake of that irregularity, and the inaccuracy of the observation of May 3, may have affected the elements.

This might have been avoided, by combining a greater number of observations in correcting the approximate elements, but the advantage would not, probably, have repaid the additional labour.

The remark on these latter observations, must not be understood to imply, in the smallest degree, a defect in observing; on the contrary, the general exactness of the observations appears highly creditable to the observers, and requires no

allowance for the unavoidable difficulties under which they must have observed.

Note.—The computations relative to the comet observed by Captain HALL, were finished in the middle of October last, *and the results immediately sent* to London for the purpose of being laid before the Royal Society. The second part of the Transactions for 1821, did not reach me till after the communication had been read at the Royal Society. In that second part, I was much surprised to find the elements of the same comet computed by Mr. RUMKER, from the observations made by Dr. OLBERS, before the passage through perihelion.

Subsequently, the "Conn. des Tems," for 1824 reached me, which contains Observations made at Paris, and Elements by M. NICOLLET; also a notice of the comet having been observed in several places of Europe. It certainly is highly creditable to those observers who discovered, under very difficult circumstances, the comet in its approach to the sun.

By the addition of Captain HALL's observations after the passage through perihelion, we are enabled to obtain very exact elements.

The errors of my Elements, when applied to observations before perihelion, and the errors of Mr. RUMKER's and of M. NICOLLET's Elements, when applied to Captain HALL's observations after perihelion, are considerable:

Therefore, I have farther corrected my Elements, by using Dr. OLBERS' observations of January 30, with those of Captain HALL of April 8, and May 3.

The new elements (C) are

Perihelion distance	,091677		
Time of passage through perihelion - - -	March 21,	11h. 11m. 48s.	Mean time Greenwich.
Inclination - - - -		73° 34′ 53″	
Node - - - - -		48 42 18	
Place of perihelion - -		239 30 33	

Motion retrograde.

Errors of the Elements (C).

	Error in Longitude.	Error in Latitude.	Observers.
	′ ″	′ ″	
Jan. 21	+ 0 18	— 0 12	M. NICOLLET, Paris.
30	+ 1 6	+ 0 13	Dr. OLBERS, Bremen.
Feb. 9	— 0 28	— 0 44	M. NICOLLET.
Mar. 6	— 2 17	— 0 57	Dr. OLBERS.
April 8	+ 0 18	— 0 45	Captain HALL, Valparaiso.
20	+ 0 11	— 0 10	Ditto.
May 3	— 0 49	+ 0 12	Ditto.

Between January 21, and May 3, the comet described above 300° about the sun: consequently, as a parabola represents the observations with so much exactness, it follows, that the period of its revolution must be considerable.

J. B.

Observatory, Trinity College, Dublin,
Feb. 25, 1822.